Massless Majorana fermion on the domain wall

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We study the domain-wall formalism with additional Majorana mass term for the unwanted zero mode, which has recently been proposed for lattice construction of 4D $\mathcal{N}=1$ super Yang-Mills theory without fine-tuning. Switching off the gauge field, we study the dispersion relation of the energy eigenstates numerically, and find that the method works for reasonable values of Majorana mass. We point out, however, that a problem arises for too large Majorana mass, which can be understood in terms of the seesaw mechanism.

1. Introduction

Supersymmetry is difficult to realize on the lattice. This is not so surprising since the lattice regularization breaks the translational invariance, which forms a subgroup of the supersymmetry. As the translational invariance is restored in the continuum limit, we can restore supersymmetry in the continuum limit. But the price we have to pay for the latter is that we need finetuning in general. For 4D $\mathcal{N}=1$ supersymmetric Yang-Mills (SYM) theory, one can use the Wilson-Majorana fermion for the gaugino and recover supersymmetry in the continuum limit by fine-tuning the hopping parameter to the chiral limit [1]. Some numerical works have been started along this line [2]. Fine-tuning is a hard task, however, as is known in the numerical studies of the chiral limit in QCD, and a method without fine-tuning is highly desired.

The overlap formalism [3] can be used for this purpose, since it preserves exact chiral symmetry on the lattice. The problem here is that the formalism is not suitable for numerical simulation as it stands. A practical proposal made by Ref. [4] is to use the domain-wall formalism [5,6], and to decouple the unwanted zero mode by adding Majorana mass term for it.

In this article, we examine whether this approach really works when the gauge field is switched off as a first step. We diagonalize the

Hamiltonian of the system numerically and study the dispersion relation of the energy eigenstates for various additional Majorana mass.

2. The model

The action of the model consists of two parts:

$$S = S_0 + S_{\text{mass}}. (1)$$

 S_0 is given by

$$S_{0} = \sum_{s,t} \{ \bar{\xi}_{s} \delta_{st} \sigma_{\mu} \partial_{\mu} \xi_{t} + \bar{\eta}_{s} \delta_{st} \bar{\sigma}_{\mu} \partial_{\mu} \eta_{t} + \bar{\xi}_{s} \mathcal{M} \eta_{t} + \bar{\eta}_{s} \mathcal{M}^{\dagger} \xi_{t} \},$$

$$(2)$$

where $\mathcal{M} = \delta_{s+1} t - \delta_{st} \left(1 - M - \frac{1}{2}\Delta\right)$. With this action, one obtains a right-handed massess Weyl fermion and a left-handed one localized at the boundaries of the fifth direction $s = 0, N_s$, respectively. In the $N_s \to \infty$ limit, the chiral symmetry is exact and we end up with one massless Dirac fermion [6]. For finite N_s , the chiral symmetry is broken but this breaking vanishes exponentially with increasing N_s [7].

When we consider 4D $\mathcal{N}=1$ SYM theory, the fermion is in the adjoint representation, which is real, and hence the Weyl fermions can be viewed as Majorana fermions, one of which has to be decoupled, while the other kept exactly massless. Let us identify the zero mode in ξ as the massless Majorana fermion we want, namely the gaugino.

In order to decouple the unwanted zero mode in η , we introduce the additional term $S_{\rm mass}$ to the action. Note that since the fermions are in a real representation, we can introduce Majorana mass term for each Weyl fermion, ξ , η , independently, without violating the gauge symmetry. There is a variety of choice for the $S_{\rm mass}$, but as the simplest one, we restrict ourselves to the Majorana mass term for the η localized at the boundary $s=N_s$ [4]:

$$S_{\text{mass}} = m \sum_{s} \left(\eta_s^T \sigma_2 \eta_s + \bar{\eta}_s \sigma_2 \bar{\eta}_s^T \right) \Big|_{s=N_s}.$$
 (3)

3. The dispersion relation

We derive the Hamiltonian of the system from the action and diagonalize it numerically. Since we have switched off the gauge field, the system is translationally invariant, and therefore we can partially diagonalize the Hamiltonian by working in the momentum basis. We calculate numerically the energy eigenvalues for fixed three-dimensional momentum **p**.

For moderate Majorana mass we have one massless Weyl fermion localized at s=0 as expected. Figure 1 shows the energy of ξ and η as a function of p_x where $p_y=p_z=0$ when the Majorana mass is 0.2. One can see that the ξ has a linear dispersion relation, while the η has a mass gap.

One might think that larger Majorana mass only results in larger mass for η without any problem, but this is not the case. Figure 2 shows the dispersion relation for large Majorana mass m=1000. One can see that although the ξ remains massless, the doublers of ξ become massless also.

In order to clarify the situation, we plot in Fig. 3 the mass of the next lightest mode at $\mathbf{p}=\mathbf{0}$ as well as that of the doublers of ξ as a function of the Majorana mass. The doublers have mass of the order of the cutoff for m<1.0, but for m>1.0, the mass decreases with increasing Majorana mass as $\sim 1/m$. The doublers with many π 's in the momentum component are heavier than those with less π 's. The mass of the next lightest mode at $\mathbf{p}=\mathbf{0}$ grows linearly as the Majorana mass increases, but saturates for m>0.5. By

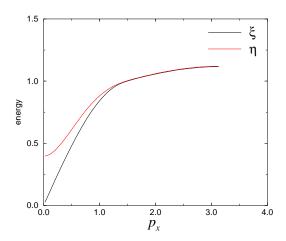


Figure 1. The dispersion relation of ξ and η for m = 0.2

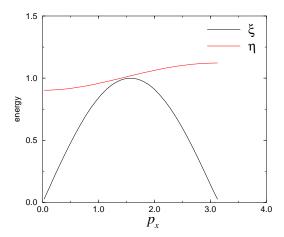


Figure 2. The dispersion relation of ξ and η for m = 1000

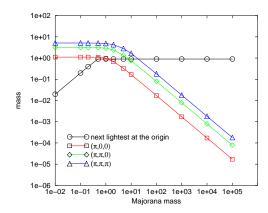


Figure 3. The mass of the next lightest mode at $\mathbf{p} = \mathbf{0}$ as well as that of the doublers of ξ as a function of the Majorana mass.

looking at the wave functions, we find that this mode consists mostly of η for m < 0.5, and becomes a mixture of ξ and η for m > 0.5.

4. Interpretation of the result for the large Majorana mass

The behavior of the doublers for large Majorana mass we have seen in the previous section can be understood in terms of the seesaw mechanism. The situation considered is the case in which Dirac mass and Majorana mass term coexist. When we diagonalize the mass matrix of the fermion, a very small eigenvalue appears when the Majorana mass is much larger than the Dirac mass.

In fact, the doublers have the two types of mass term in our model. The Dirac mass term comes from the Wilson term in (2) and can be written as

$$S_{\text{Dirac}} = 2n \sum_{s} (\bar{\xi}_s \eta_s + \bar{\eta}_s \xi_s), \tag{4}$$

where n is the number of π 's in the momentum component of the doubler. Together with the Ma-

jorana mass term which comes from (3), we have the following mass matrix for the doublers.

$$\left(\begin{array}{cc}
0 & 2n \\
2n & m
\end{array}\right).$$
(5)

The eigenvalues of this matrix for $m \gg n$ are given by $\lambda \simeq m, \frac{4n^2}{m}$. The second one is almost zero for $n/m \ll 1$, which explains why the doublers become very light for large Majorana mass.

5. Summary

We examined whether the proposal for decoupling the unwanted zero-mode in the domain-wall approach by adding the Majorana mass term for it works when the gauge field is switched off. Above all, we clarified what values we should take for the Majorana mass to be added. We observed the desired dispersion relation for moderate values of the Majorana mass, which means that the approach is promising. We pointed out, however, that for too large Majorana mass, the doublers of the desired Majorana fermion become very light. We gave a natural explanation for this phenomenon in terms of the seesaw mechanism. We can also obtain the above result by the analysis of the fermion propagator [8], which will be reported elsewhere [9]. Our next task is of course to switch on the gauge field, which we would like to report in future publications.

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